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★Projectors on intersections of subspaces. (English summary)

Infinite products of operators and their applications, 41–50, *Contemp. Math.*, 636, *Israel Math. Conf. Proc.*, *Amer. Math. Soc.*, *Providence*, *RI*, 2015.

The paper deals with constructions of orthogonal projectors on intersections of subspaces in finite-dimensional $\mathbb{C}$-linear spaces. Projectors are constructed as functions of orthogonal projectors on the subspaces and their orthogonal complements using SVD (singular value decomposition) and matrix exponentials.

The paper consists of 5 sections. Linear spaces over $\mathbb{C}$ are considered.

1. Introduction: Review of previous results concerning projectors on intersections of subspaces. Short description of results of the paper.

2. Principal angles: Definitions of reciprocal vectors and principal angles between two subspaces $\mathbf{L}$ and $\mathbf{M}$. Historical review.

3. $P_{\mathbf{L}\cap\mathbf{M}}$ and the singular value decomposition of $P_{\mathbf{L}}P_{\mathbf{M}}$: Theorem 3.2. Expression of $P_{\mathbf{L}\cap\mathbf{M}}$ and $\|(P_{\mathbf{L}}P_{\mathbf{M}})^n - P_{\mathbf{L}\cap\mathbf{M}}\|$ using the SVD factors of $P_{\mathbf{L}}P_{\mathbf{M}}$. An example is given.

4. Dual representations: Lemma 4.1. Let P_i be subspaces and let P_i and $P_i^{\perp}$ be orthogonal projectors in subspaces and their orthogonal complements. Then

$$\bigcap_{i=1}^{n} \mathbf{L}_i = N\left(\sum_{i=1}^{n} \lambda_i P_i^{\perp}\right).$$

Corollary 4.2. $P_{\mathbf{L}_1\cap\cdots\cap\mathbf{L}_n} = E - Q^{\dagger}Q$, where $Q = E - \frac{1}{m}\sum_{i=1}^{n} P_i$.

5. Projectors as limits of exponentials: Corollary 5.3. $P_{\mathbf{L}_1\cap\cdots\cap\mathbf{L}_n} = \lim_{t\to\infty} e^{-Qt}$, where $Q = \sum_{i=1}^{n} \lambda_i P_i^{\perp}$, $\lambda_i > 0$.

{Reviewer's remarks: (1) The paper seems to have been written by a qualified linear algebraist; the language is dense and precise. Notations are traditional and clear. (2) New constructions of projectors on subspace intersections are obtained using SVD and matrix exponentials. The results seem to have computational value. (3) Computational complexity reduction benefits compared to the previous results (especially formula (1.5)) are not given.}

{For the collection containing this paper see MR3155357}

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